

**Detection of MIMO Systems using MMSE-Based Lattice-Reduction Using Front
End Design**

Sampath

sampathece04@gmail.com

Abstract

In recent publications the use of lattice-reduction for signal detection in multiple antenna systems has been proposed. In this paper, we adopt these lattice-reduction-aided schemes to the MMSE criterion. We show that an obvious way to do this is infeasible and propose an alternative method based on an extended system model, which in conjunction with simple successive interference cancellation nearly reaches the Performance of maximum-likelihood detection. MMSE detection, lattice-reduction, wireless communication. The front-end is a crucial component in modern wireless communication systems. For SISO systems minimizing the front-end noise factor is optimal, and this also applies to MIMO systems with i.i.d. noise. However, for compact MIMO receivers that may exhibit spatially correlated noise, e.g., through antenna mutual coupling, the optimal design procedure is not clear. In this paper we develop MIMO low-noise design principles from a communication theory perspective by deriving generalizations of SNR and noise factor from various MIMO communication schemes. As one result, we are able to derive optimal matching networks for a bank of uncoupled amplifiers.

Keywords: MIMO systems, antenna arrays, noise figure, receivers, channel capacity.

Introduction

SISO (single input, single output) refers to a wireless communications system in which one antenna is used at the source and one antenna is used at the destination. SISO is the simplest antenna technology. In some environments, SISO systems are vulnerable to problems caused by multipath effects. When an electromagnetic field is met with obstructions such as hills, canyons, buildings, and utility wires, the wave fronts are scattered, and thus they take many paths to reach the destination. The late arrival of scattered portions of the signal causes problems such as fading, cut-out and intermittent reception (picket fencing). In a digital communications system, it can cause a reduction in data speed and an increase in the number of errors. MIMO (multiple inputs, multiple outputs) is an antenna technology for wireless communications in which multiple antennas are used at both the source and the destination. The antennas at each end of the communications circuit are combined to minimize errors and optimize data speed. In conventional wireless communications, a single antenna is used at the source, and another single antenna is used at the destination. In some cases, this gives rise to problems with multipath effects. When an electromagnetic field is met with obstructions such as hills, canyons, buildings, and utility wires, the wave fronts are scattered, and thus they take many paths to reach the destination. The late arrival of scattered portions of the signal causes problems such as

fading, cut-out and intermittent reception. In digital communications systems such as wireless Internet, it can cause a reduction in data speed and an increase in the number of errors. The use of two or more antennas, along with the transmission of multiple signals (one for each antenna) at the source and the destination, eliminates the trouble caused by multipath wave propagation, and can even take advantage of this effect. MIMO technology has aroused interest because of its possible applications in digital television (DTV), wireless local area networks (WLANs), metropolitan area networks (MANs), and mobile communications.

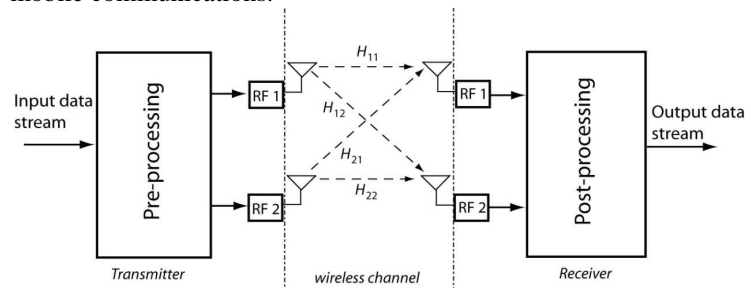


Fig 1.0 transmitting the data

Signal-To-Noise Ratio

Signal-to-noise ratio, or SNR, is a measurement that describes how much noise is in the output of a device, in relation to the signal level. Signal-to-noise ratio is defined as the power ratio between a signal and noise. It can be derived from the formula

$$SNR = P_{\text{signal}}/P_{\text{noise}}$$

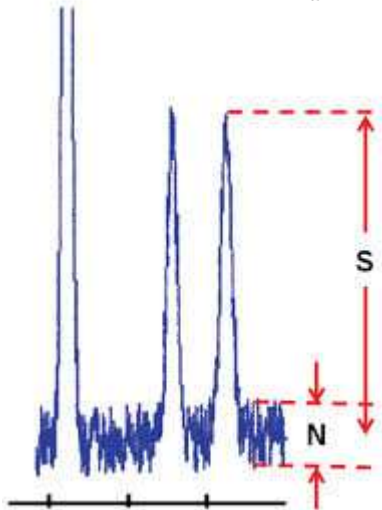


Fig 2.1 Receiver model of MIMO

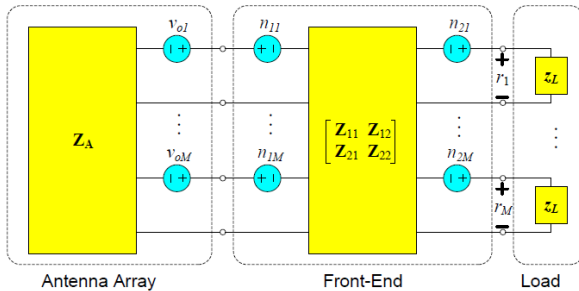


Fig 2.2 Front end model of MIMO

MIMO receiver model have M transmitting antennas and N receiving antennas. In general the receiving antennas are used to amplify the signal and also convert analog into digital.

Detection Algorithms

A. Common ZF and MMSE Detection Algorithms

In a zero-forcing (ZF) detector the interference is completely suppressed by multiplying the receive signal vector \mathbf{x} with the Moore-Penrose pseudo-inverse of the channel matrix

$$\mathbf{H}^+ = \mathbf{H}^T \mathbf{H} \mathbf{H}^T$$

The decision step consists of mapping each element of the \mathbf{H}^+ iter output vector

$$\tilde{\mathbf{s}}_{ZF} = \mathbf{H}^+ \mathbf{x} = \mathbf{s} + (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{n}$$

quantization, which in case of M-QAM (after proper shifting and scaling) corresponds to a simple rounding operation and (if necessary) clipping to the allowed range of values. For an orthogonal channel matrix, ZF is identical to ML. However, in general ZF leads to noise amplification, which is especially observed in systems with the same number of transmit and receive antennas.

Antenna Array

An antenna array is a set of 2 or more antennas. The signals from the antennas are combined or processed in order to achieve improved performance over that of a single antenna. Let $\mathbf{v}, \mathbf{i} \in \mathbb{C}^M$ denote the complex-baseband voltage across and current flowing into the antenna array terminals, respectively. The circuit equation for the array is $\mathbf{v} = \mathbf{Z}\mathbf{A}\mathbf{i} + \mathbf{v}_o$, where $\mathbf{Z}\mathbf{A}$ is the antenna impedance matrix and \mathbf{v}_o is the open-circuit (induced) voltage. Throughout this paper we assume all impedance matrices are nonsingular, i.e., every nonzero current input produces a nonzero terminal voltage. When necessary, we shall express impedance matrices (uniquely) as $\mathbf{Z} = \mathbf{R} + j\mathbf{X}$, where $\mathbf{R} \in \mathbb{R}^{2 \times 2}$ ($\mathbf{Z} + \mathbf{Z}^\dagger$), $\mathbf{X} \in \mathbb{R}^{2 \times 2}$ ($\mathbf{Z} - \mathbf{Z}^\dagger$), and the \dagger superscript denotes the conjugate transpose. For one-port devices this reduces to $z = r + jx$, where r and x are the device resistance and reactance. Most prior studies in MIMO receivers have been restricted to uncoupled amplifiers. Here we take a more general viewpoint and assume the front-end may have arbitrary coupling and noise correlation between its ports. These circuit quantities are related by

$$\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}$$

The minimum mean square error (MMSE) detector takes the noise term into account and thereby leads to an improved performance. As shown in [6], [7], MMSE detection is equal to ZF with respect to an extended system model. To this end, we define the $(n+m) \times m$ extended channel matrix \mathbf{H} and the $(n+m) \times 1$ extended receive vector \mathbf{x} by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_m \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} \mathbf{x} \\ \mathbf{0}_{m,1} \end{bmatrix}$$

Then, the output of the MMSE filter can be written as

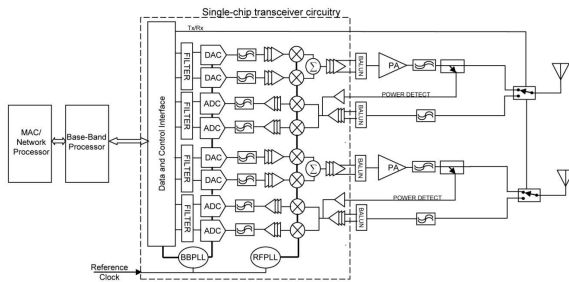
$$\begin{aligned} \tilde{\mathbf{s}}_{MMSE} &= (\mathbf{H}^T \mathbf{H} + \sigma_n^2 \mathbf{I}_m)^{-1} \mathbf{H}^T \mathbf{x} \\ &= (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{x} = \mathbf{H}^+ \mathbf{x}, \end{aligned}$$

B. Lattice Reduction aided Linear Detection

As already mentioned, linear detection is optimal for an orthogonal channel matrix. Now, with $\tilde{\mathbf{H}}$

= HT and the introduction of $z = T^{-1}s$ the receive signal vector can be rewritten as

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{H}\mathbf{T}\mathbf{T}^{-1}\mathbf{s} + \mathbf{n} = \tilde{\mathbf{H}}\mathbf{z} + \mathbf{n}.$$



Note that $\mathbf{H}\mathbf{s}$ and $\tilde{\mathbf{H}}\mathbf{z}$ describe the same point in a lattice, but the LLL-reduced matrix $\tilde{\mathbf{H}}$ is usually much better conditioned than the original channel matrix \mathbf{H} . For $s \in \mathbb{Z}^M$ we also have $z \in \mathbb{Z}^M$, so \mathbf{s} and \mathbf{z} stem from the same set. However, for M-QAM, i.e. $s \in \mathbb{Z}^M$, the lattice is \mathbb{Z}^M and the domain of \mathbf{z} differs from \mathbb{Z}^M . This is illustrated in Fig. 2 for 16-QAM, one transmit antenna ($M = 2$) and a transformation matrix $\mathbf{T} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Conclusion

In this paper we derived optimal front-end design principles from a communication theory perspective by examining the form of several MIMO communication schemes. We investigated several detection schemes for multiple antenna systems making use of the lattice-reduction algorithm proposed by Lenstra, Lenstra and Lov'asz. We showed that the straightforward way to perform MMSE detection after lattice-reduction does not yield satisfying results. Through this approach vector channel generalizations of SNR and noise factor were developed, and a sufficient condition for low-noise design was derived. The result applies to a larger class of front-ends than is typically considered in the communications literature, but may be readily applied to practical problems such as amplifier matching. In fact, with the developed theory we were able to derive optimal matching networks that were only conjectured to be optimal in previous studies. Through numerical simulations we verified this result, and also demonstrated the need for alternative noise measures such as the noise factor matrix by showing that the two-port noise factor may not be able to accurately predict performance in the presence of coupled antennas. Instead, we proposed a new method, where LR is applied to an extended system model. In conjunction with successive interference cancellation, this strategy nearly leads to maximum-likelihood performance. Furthermore, we analyzed the impact of a sorted QR decomposition on the LLL algorithm and demonstrated that SQRD can

dramatically decrease the computational effort. Thus, we arrived at a near-optimum detector with very low complexity.

References

- [1] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," in *IEEE Proc. ISSSE*, Pisa, Italy, September 1998.
- [2] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, "Closest Point Search in Lattices," *IEEE Trans. on Information Theory*, vol. 48, no. 8, pp. 2201-2214, August 2002.
- [3] H. Yao and G. Wornell, "Lattice-Reduction-Aided Detectors for MIMO Communication Systems," in *IEEE Proc. Globecom*, Taipei, Taiwan, November 17-21 2002.
- [4] C. Windpassinger and R. F. H. Fischer, "Low-Complexity Near-Maximum-Likelihood Detection and Precoding for MIMO Systems using Lattice Reduction," in *IEEE Proc. ITW*, Paris, France, March 2003.
- [5] B. Hassibi, "An Efficient Square-Root Algorithm for BLAST," in *IEEE Proc. ICASSP*, Istanbul, Turkey, June 2000, pp. 5.9.
- [6] D. Wubben, R. Bohnke, V. Kuhn, and K. D. Kammeyer, "MMSE Extension of V-BLAST based on Sorted QR Decomposition," in *IEEE Proc. VTC-Fall*, Orlando, Florida, USA, October 2003.